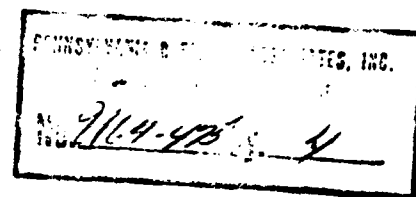


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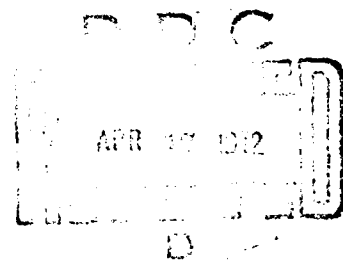
FINAL REPORT:  
INVESTIGATION OF THE COMPILATION  
OF DIGITAL MAPS

Pennsylvania Research Associates, Inc.  
133 South 36 Street  
Philadelphia, Pennsylvania 19104

3 February 1964

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U.S. NAVAL TRAINING DEVICE CENTER  
Port Washington, L.I., New York

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NATIONAL TECHNICAL  
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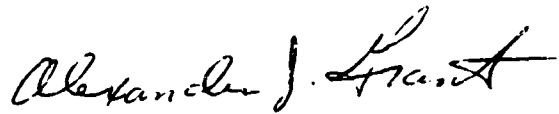
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FOREWORD

This report is developed under amended contract N61339-1025. This study was directed toward a firmer understanding of the information content of maps and their digital representation. This work in the compilation of digital maps is motivated by the need for such maps in radar simulation.

This report contains a number of ad hoc investigations into map encoding. This is very appropriate but gives the impression that actually encoding the maps in a reasonably practical way is a very serious problem, even if one has solved the storage and computation problems. The studies do yield a certain amount of map statistics which are based on a very small sample.



Project Engineer  
Naval Training Device Center

The findings in this report are not to be construed as an official Department of the Navy position.

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## I. INTRODUCTION

A preceding program under Contract N61339-1025 between the U.S. Naval Training Device Center and Pennsylvania Research Associates, Inc., "Investigation of Digital Techniques for Radar Land Mass Simulation" (Refs. 1, 2, and 3), illustrated the necessity for compressing the data used to describe natural terrain and cultural complexes. That program developed the contour tracing method for representing a function of two or more variables (such as a map of terrain height or radar reflectivity) so that it could be manipulated, stored, processed, and transmitted by digital data handling equipment.

It was recognized, however, that there was no suitable measure of the amount of data needed for digital representation of a map, and thus that there was no means of determining how efficient the contour tracing method was, as far as data compression is concerned. In addition, that method, while achieving significant compression ratios (three orders of magnitude over straightforward tabulation of terrain height and reflectivity), required significant complexity in the computational equipment for reconstructing terrain profiles from the stored data.

Accordingly, the present program, "Investigation of the Compilation of Digital Maps", was initiated under the same contract as a preliminary study of the form and amount of data needed to represent in an optimal manner a pictorial map, a portion of the earth's surface, or indeed any function of two or more dimensions. The body of knowledge compiled under the study of information theory was to be applied to develop a measure of the information content of a map against which any given encoding or representation scheme could be compared. Encoding techniques were to be derived and evaluated on the basis of this theory, and characteristics of actual maps were to be studied to determine how and where to apply these techniques.

This program, then, is a preliminary investigation of efficient methods and techniques for preparing and using map information in digital form, not necessarily limited to maps prepared as the input data for radar land mass simulators. Whereas pictorial or analog maps are readily readable by a human, digital processing of map data is severely hampered by the

awkwardness of coupling a conventional pictorial map to a digital data processor for storing, selecting, forwarding, and updating of the map.

The usual advantage of unlimited and variable precision favors consideration of digital map representation over analog or pictorial methods. The position of a benchmark may be very carefully drawn on a pictorial map, the purpose of the benchmark being to serve as a reference or fiducial point for precise calculations. However, the position of an "X" on a piece of paper is subject to random and systematic errors in observation and reproduction that can be reduced only by proportionately increasing the scale of the map. Were this information recorded in digital form, the precision of the digital word expressing the coordinates of the point can be as great as necessary for that point, without the necessity of retaining that same precision for the remainder of the information in the map.

Other advantages are readily apparent. One may conceive of a universal or master digital map of some portion of the globe, in which the data is stored in a high-density medium such as magnetic tape. The information may be stored to as great a precision as desired. Should a map be required for a given area of terrain or at a particular scale factor, the information is selected by computer from the master model contained in the storage medium. A portion of the information on a map must frequently be corrected and updated as revised information becomes available, and such revisions should be made without the necessity for having to recompile the entire map. The convenience of manipulating reels of magnetic tape (or a similar digital storage medium) appears to have significantly greater long-term potential than does special-purpose equipment to manipulate photographs, lithographs, or relief models.

This report discusses the applicability of various concepts and techniques of information theory to this problem. The gradient vector method of digital representation is derived on the basis of that theory. It is applied, along with the contour tracing method, to sample portions of the earth whose data were obtained in the form of pictorial topographic maps. It is concluded that the contour tracing method, without interpolation, is useful for maps in which planimetric aspects predominate and the functions to be mapped are discrete -- such as radar reflectivity. On the other hand, the gradient vector

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method is found to be more efficient for maps in which the hypsographic aspects predominate and the functions to be mapped are continuous -- such as terrain height. Finally, certain recent advances are shown to have bearing on the problem, and avenues of approach are suggested which appear fruitful for further investigation.

## II. INFORMATION CONTENT OF A MAP

While several methods have been published for compressing the data required to represent a topographic map, television picture, or other function of several variables, these methods may be compared only on a relative basis. That is, there is no absolute measure of the extent to which the data could be compressed. It may be recognized that the body of knowledge called information theory, or communication theory, deals with this subject -- the theoretical minimum amount of data required to convey certain information about a physical process. However, conventional theory is complete and rigorous only in the case of discrete functions of one variable, plausible and useful in the case of continuous functions of one variable, and virtually nonexistent in the case of continuous functions of many variables. (See Bibliography)

This section is a sketch of the applicable portions of the body of knowledge developed primarily by Shannon and called information or communication theory, as well as a description of a new method for describing the information content of a function of several variables in terms of the (more conventional) information content of functions of one variable. This material does not purport to be a treatise on information theory, which subject is now covered in numerous textbooks and monographs, but attempts to organize the concepts and techniques of that theory having direct applicability to development of a useful measure of the information content of a (continuous, multi-dimensional) map. In particular, the gradient vector method of reducing the dimensionality is derived, based upon these concepts and techniques.

A map may be considered the superposition of many continuous functions of (at least) two variables. Subject to accuracy, error, or fidelity criteria that may be found pertinent to the problem of map representation, it is desired to establish a measure of the minimum amount of data required to represent an arbitrary selected map, in terms of statistical knowledge about the entire collection of maps from which the selected one is drawn. Thus it is desired to measure the information content of a map (or of any function of two or more variables). This will be the minimum amount of data required to convey the information in the map, to the accuracy, error, or fidelity limits that may be established.

Should such a measure be found, it will determine the extent of data compression that may be achieved by any scheme of map representation. It may also suggest means for approaching that compression and serve as a standard against which to compare different schemes that may be devised.

#### A. Existing Theory and Its Generalization

Presented here is a summary of pertinent prior work, along with discussion of newer approaches to the problem of measuring the information content of a continuous (noise-free) multi-dimensional function. Significant terms and concepts are underlined where they appear for the first time in this text. Published literature defines and discusses such terms and concepts in far more detail than is suitable herein, and the Bibliography of this report contains numerous references thereto.

A map is a representation of portions of the real world -- an abstraction of reality. The real world, or an existing pictorial map thereof, may be considered a message source, and a digital map is then an encoded message. This encoded message will be handled by a transmission channel -- the physical devices that store and process the data representing portions of the real world. This encoded message must then simultaneously be an adequately faithful representation of the original map or real world, and be readily handled by physical devices with economically limited storage and processing complexity. In other words, the encoding must not destroy needed information, but must compress the data describing the real world so that it may be handled by channels with finite capacity.

The problem is thus to find the entropy (the average amount of uncertainty removed upon reception of a message) of this message source and then to devise an encoding (representation) scheme to match it, under suitable restrictions on form and amount of allowable error in reconstruction of the original message. In the classical case of a discrete, stationary message source, consideration is first made of the relative probabilities of the different possible sequences of symbols, or messages, that may be transmitted. It can be shown that a useful measure of entropy  $H$  is the average logarithm of the probabilities  $p_i$  of all the possible symbols, or  $H = - \sum p_i \log p_i$ . Entropy per symbol thus represents the average amount of uncertainty removed when an arbitrary symbol is specified in

a sequence of symbols. The information conveyed by a message is given by the reduction in entropy that occurs when the information is transmitted, or when the probabilities are changed.

If logarithms are taken to the base two, the entropy is measured in bits, derived from binary digits. This really means the number of binary decisions to be specified, on the average, to convey the message, but it is usually taken to mean the number of digits by which the average message may be represented. It may be shown that entropy -- potential uncertainty -- is maximized when the symbols are uniformly distributed. That is, if  $p_i = 1/N$  for all  $N$  symbols in the alphabet, the entropy reduces to  $\log_2 N$  bits per symbol, or  $\lg N$ .

However, if the probabilities of the various symbols from the alphabet are not identical, the entropy of the source is reduced, and an advantage in the number of symbols to convey the information may be obtained by coding the messages. That is, sequences of symbols corresponding to the original messages may be represented by sequences of symbols -- not necessarily from the same alphabet -- corresponding to the encoded messages. Individual symbols from the original message, as well as strings of such symbols, are mapped (in the mathematical sense) onto symbols or strings of the code, from which coded message the original message is supposedly reconstructable. The object of coding is usually to make the length or probability of the sequence of encoded messages more nearly uniform, with the attendant decrease in the number of symbols and/or size of the alphabet required to convey the messages.

The "best coding", or the minimum number of bits that can be obtained by any possible method of coding, is given by the entropy of the source. This bound can be approached as a limit for arbitrarily long sequences of symbols to be encoded and then, of course, decoded. A famous theorem of Shannon states that a channel of capacity equal to the entropy of the source can transmit, or represent, all messages with error as small as desired by the use of a sufficiently complex coding scheme. Huffman (see Bibliography) has devised a code for which the number of bits per encoded message is not more than one greater than the entropy of the source, measured in bits per original message. For example, consider a binary source possessing redundancy to the extent that its entropy is only  $10^2$  bits per symbol. If each symbol of the original sequence is

represented by one binary symbol in the encoded sequence (no coding), the encoded sequence requires one bit per symbol of the original. However, if strings of 100 bits of the original sequence are represented by a suitable code, the encoded sequence can be shown to require between one and two bits per 100 symbols of the original. Furthermore, it can be shown that, in the sense of data compression, this is the best coding scheme of finite length. The cost of this data compression lies in the additional coding/decoding and storage required to handle such long strings. This processing and storage cost is then a trade-off with the cost of storing the messages in uncoded form.

For a map, in which a discrete function is treated as a sequence of symbols in two dimensions, the measure of entropy will be in bits per two-dimensional symbol. Instead of the message being transmitted as a function of time, it is transmitted as a function of space. The entropy of a message source is measured not in bits per unit time (obviously related to bits per symbol by the rate of symbol transmission), but in bits per unit area. The information conveyed by a map is thus given by the change in entropy, or the reduction in uncertainty, or the trend toward nonuniform probabilities, attendant to the specification or transmission of the map.

When the foregoing discrete theory is generalized to the theory of continuous functions, the terminology changes somewhat. Instead of symbols from an alphabet, the source is considered to transmit values within a range, and the entropy is measured in bits per unit time (or area) without reference to the rate of transmission of identifiable symbols. The discrete probability distribution of symbols is replaced by a continuous distribution of values.

To derive the entropy of a continuous function, consider the values to be grouped, or the range quantized, into intervals  $y_i$  of fixed width  $\Delta y$ . Then the entropy corresponding to this distribution is given by

$$H(Y) = - \sum_i p(y_i) \Delta y \left[ \lg p(y_i) + \lg \Delta y \right].$$

Since  $\sum_i p(y_i) \Delta y = 1$  for any probability distribution  $p(y)$ , the entropy becomes  $H(Y) = - \sum_i p(y_i) \lg p(y_i) \cdot \Delta y + \lg \frac{1}{\Delta y}$ . As the quantization interval  $\Delta y$  is

decreased and the number of such intervals is increased accordingly, the second term  $\lg 1/\Delta y$  becomes larger. The first term is the entropy with respect to this constant of quantization, and in the limit this becomes the integral  $\int_{-\infty}^{\infty} p(y) \lg p(y) dy$ . Thus the entropy of a continuous function is meaningful only with respect to a stated quantization interval. But for most purposes, the entropy is treated as consisting of the relative entropy, or the integral alone, since the quantization constant is independent of the probability distribution. The difference of two entropies -- the information transmitted by changing the probability distribution -- is of course independent of the (common) quantization constant.

The values taken by the function whose distribution is  $p(y)$  may be the amplitudes of its waveform at successive instants of time. If each instant of time is then considered independently and the range of amplitude is  $\pm A$ , it may be shown that the uniform amplitude distribution affords maximum entropy, which maximum becomes  $\log 2A$ . On the other hand, if the range is unbounded but the mean square value is held constant at  $\sigma^2$ , it may be shown that the gaussian distribution  $p(y) = 1/\sqrt{2\pi}\sigma \exp - y^2/2\sigma^2$  has maximum entropy, which maximum becomes  $\log \sqrt{2\pi e} \sigma$ .

Since communication systems usually have a limit on maximum power, or on rms amplitude  $\sigma$ , coding of messages into waveforms having gaussian distributions of amplitude is usually attempted. In fact, Shannon (see Bibliography) shows that this represents the optimum immunity to gaussianly distributed additive interference, or noise. However, the noise due to quantization of a continuous waveform represents essentially a uniform distribution over the range plus or minus half a quantization interval. This maximum amplitude error  $\pm \Delta y/2$  is of course related to the quantization constant  $\log 1/\Delta y$  and is the criterion of accuracy, error, or fidelity usually considered. But the real world has no such inherent quantization of values; what is a suitable criterion on the basis of which to characterize the entropy of a map?

So far, the symbols of a discrete alphabet or values from a continuous range have been taken to be independent. It can be shown that any probabilistic constraints between symbols or values at different locations along the sequence, or waveform, serve only to reduce the entropy of the source. Such

constraints are called contextual dependencies, since the probability distribution of a symbol depends on the influence of one or more other symbols in the sequence. If the domain of time or distance over which a function is defined is continuous, regardless of whether the function takes on discrete or continuous values within its range, a complete lack of contextual dependency would imply independent instants of time or distance, and thus the function would consist merely of a series of impulses. But real physical systems tend to have smoothness or continuity in general. Further, a series of impulses would have a uniform power spectrum or distribution of power as a function of frequency, whereas real physical systems tend to be bandlimited or have their power distributed in a finite frequency range.

The effect of intersymbol influence or bandlimiting can be expressed in part by the autocorrelation function, whose properties are well known. It gives the correlation coefficient between two symbols of a sequence, or values of a waveform, in terms of their respective locations in time or distance; for stationary functions only the relative location is important. The correlation between adjacent symbols in natural printed language is expressed by the digram structure; trigram and higher order probabilistic constraints serve further to reduce the entropy from that obtained by considering the symbols as being independent. By the Wiener-Khintchine relation, the autocorrelation function and power spectrum of any arbitrary waveform are Fourier transform pairs.

Quantization was introduced as a scheme for reducing the continuum of values of a function to a (denumerably infinite) set of quantization intervals. Similarly, the continuum of locations of a domain can be reduced to a (denumerably infinite) set by sampling a waveform. Appeal to a sampling scheme is equivalent to looking at the sampled function thru an aperture described in the frequency domain. The Fourier transform of this aperture describes the effect in the time domain, and these effects must be compensated in an attempt to reconstruct the original continuous function.

The sampling theorem in its elementary form states that a waveform which is bandlimited to some frequency  $f_0$  or below can be reconstructed from its values at sample points at a spacing of  $2/f_0$  or above, the error at any location being a monotonic nonincreasing function of the number of adjacent samples used in the reconstruction of the original value at that location. (Nonuniform spacing and/or samples

of derivatives of the waveform may be used also.) But the real world is not bandlimited unless it is filtered, or viewed thru a device of finite aperture; what is a suitable aperture shape on the basis of which to determine the number of samples required to reconstruct a map?

For continuous functions of more than one variable these concepts carry over. A waveform, instead of being a function of time, is a function of the two or more variables of generalized distance, and thus becomes a surface in the space of the appropriate number of dimensions. Contextual dependencies appear, however, in all directions, and not only along the directions chosen for the coordinate axes. For stationary functions the spatial autocorrelation function expresses in part the probabilistic constraints between values of the surface in terms of their spacing as well as their relative orientation. The multidimensional Fourier transform of the spatial autocorrelation function is the multidimensional power spectrum, and the concept of spatial frequency is introduced. Sampling theorems in many dimensions have not been well studied.

Multidimensional functions are usually reduced in dimensionality by the process of scanning. Television pictures are scanned with a rectangular raster, and coding of the resulting function (intensity) of one variable (time) may be attempted. The work of Mertz and of Kretzmer (see Bibliography) and their associates indicates that significant savings in amount of data (and thus in bandwidth) may accrue by taking advantage of the redundancies or contextual dependencies along all directions, and not just along the direction of the scanning lines. The technique of run-length coding for functions of one variable has its counterpart of contour tracing for functions of two or more variables: loci of position are described within which all points have essentially the same values of the function being represented.

A direct measure of information content for continuous multidimensional functions may be obtainable by considering not an alphabet of symbols or a range of values, but a set of functions, in the manner of Kolmogorov (see Bibliography) and his associates. The distance between two functions having a common domain is defined as the maximum absolute difference between their values at all corresponding locations in the domain. Then under a given quantization scheme, the entropy

of a source is related to the minimum number of functions required to cover the function space, and the capacity of a channel is related to the maximum number of distinguishable functions the function space may contain. The entropy corresponding to the probability distribution of a set of functions may be a useful measure of the information content of a map, and extension of this work is warranted.

#### B. The Gradient Vector Method of Reduction

Presented here is a description of a new method for the reduction of a function of many variables to many functions of one variable, in such a manner as to take account of contextual dependencies that may be found. Based on the foregoing discussion, the method illustrates that a function to be represented can be scanned so as to minimize the amount of data required to represent the scanned function, by coding to exploit the nonuniformity of the probability distribution of slope of the function to be represented. The next section illustrates the data compression that may be achieved by the gradient vector approach in a discrete embodiment.

Consider a function of many dimensions that has a gradient defined everywhere. Except for an additive constant, the function may be reconstructed at all points along an arbitrary path by means of its gradient at all such points. This path may be considered as a scan of the function. If it coincides with one of the coordinate axes in which the function is suitably expressed, the scanned function is then reconstructible along that path as a function of that one variable.

But the gradient of a function is a vector quantity, independent of the coordinate system chosen. Given the gradient in any coordinate system, the function may be reconstructed exactly in that system. The question, then, is how to encode the gradient of a function.

Obviously one may choose the coordinate system to best advantage. To reduce the amount of data of the representation, coordinates should be taken in which the greatest nonuniformity (of slope) is to be found, on the average, thus taking advantage of any known contextual dependencies. For example, a function that is essentially a tilted plane has little uncertainty at successive points. On the other hand, a function that is essentially smooth (gradient distribution clustered about one value) except for a tendency for randomness (or a lack of clustering) in one direction should have its primary coordinate

oriented in that direction. This allows the primary scanning to be represented with a larger number of bits; hence the secondary scanning lines, of which there are more, may be represented with a smaller number of bits.

Therefore, a continuous function of many dimensions may be represented as many functions of one dimension, corresponding to the coordinate axes in which the gradient is to be expressed. The effectiveness of this method, insofar as map encoding is concerned, lies in the fact that the gradient is independent of the coordinates, which may be chosen so as to minimize the total amount of data in the encoded representation.

In practice there would be a limitation on the spatial frequency to be preserved in the representation, corresponding to a limitation on the resolution that is desired, and comparable to acceptable bandlimiting in the one-dimensional case. Thus a discrete representation of the function would be obtained by sampling at known intervals along the coordinate axes. The components of the gradient at sampled points, or the finite differences between sampled values, will enable reconstruction of the function with respect to some reference point.

These differences are tabulated first along the coordinate in which greatest randomness is observed, or in which greatest entropy exists. Then, beginning with values of the function along that coordinate, differences are tabulated along the remaining coordinate in which the next largest entropy exists, and so on.

If there are just  $k$  sampling points in each of  $n$  dimensions, there are a reference point and  $k-1$  differences along the first dimension,  $k(k-1)$  differences with respect to these values along the next dimension, and so on, up to  $k^{n-1}(k-1)$  differences along the last dimension. Each successive set of differences is chosen along the dimension having less one-dimensional entropy (fewer bits per difference required on the average) but more differences. The differences along each dimension are then capable of being encoded by any suitable means. The best coding possible is given by the entropy corresponding to the distribution of slope, or differences, along that dimension. Even if all dimensions contained the same entropy, any nonuniformity of the distribution of differences leads to a saving by properly encoding the differences.

Also in practice, this encoding would probably be performed for individual regions -- portions of the function having a standardized size, shape, and orientation. Some regions would have one sequence of coordinate axes that minimizes the total number of bits of the representation, and some regions would have another sequence. The tabulation of differences for each region would therefore have to be supplemented by a listing of the sequence of coordinates along which these differences are to be arranged. However, the reference points of  $k^n$  regions could be encoded in precisely the same way as the  $k^n$  sampling points of any one region, and this saving of a small number of bits per region may balance the cost of the bits to describe the sequence of coordinates within a region.

In summary, axes may be chosen to allow optimal encoding of the gradient in each dimension; the axis along or parallel to which fewest points are tabulated is the axis aligned with the direction of maximum one-dimensional entropy. The axes need not be cartesian and in fact need not be orthogonal. In these cases, however, accuracy of representation of the original function must be examined. .

Higher orders of derivatives or differences may be used along any axis. This will take advantage not only of contextual dependencies in the slope, but also of any tendency toward smoothness or consistency in higher derivatives of the function along each axis chosen. Tabulating higher order differences at a point is of course equivalent to tabulating more points along that dimension. Such a procedure may require fewer bits for the same information and should be considered.

For a given spacing of sampling points there will be an optimum region size for any class of functions to be represented by this method. Small regions (with few points per region) will not allow effective use of any encoding scheme -- only short strings of symbols will be considered as messages, or region descriptions. Large regions (with few regions per map) will not allow effective use of sequencing the coordinate axes -- regions will comprise large areas in which one-dimensional entropy is less sensitive to orientation. Thus the quest for the optimum region size for which to use the gradient vector method of reduction, requires further investigation.

### III. EXPERIMENTAL REPRESENTATION AND RECONSTRUCTION OF MAPS

This section describes two methods of representing terrain and presents data for sample regions using these methods. The methods are:

- Straight Line Approximation to Height Contours (contour tracing)
- Height Gradients at Fixed Coordinate Grid Points (fixed point).

A description is given of the maps used for the experiments and the reasons for choosing them. The procedures for deriving the representation in binary form, convenient for digital computer manipulation, are detailed for each method.

The use of Huffman coding for the height gradients in the fixed point method is shown to reduce the number of bits required to represent a region by an order of magnitude over the contour tracing method. In addition, the maximum vertical error for the fixed point representation is almost everywhere less than for the contour tracing method. In the neighborhood of deep narrow canyons, which approach discontinuous behavior, the contour method is more accurate.

#### A. Description of Maps Used

Three portions of terrain were selected from standard topographic maps of the U. S. Geological Survey 7.5-minute series, scale 1:24,000:

- One region from Lehighton Quadrangle, Pennsylvania, contour interval 20 feet.
- Two regions from Whitneyville Quadrangle, Maine, contour interval 20 feet.
- Four regions from Sabino Canyon Quadrangle, Arizona, contour interval 20 feet.

From the lower left corner of the Lehighton Quadrangle a square 11,000 x 11,000 feet was selected. This terrain was chosen for a number of interesting features: a small creek, a railroad, a few terrain peaks, a power line, and small communities. It contains a substantial number of contours that are

irregular curves, as well as several relatively linear curves, over a height variation between 520 and 1572 feet.

From the center of the Whitneyville Quadrangle, a rectangle 20,000 x 10,000 feet was selected. This terrain comprises two regions of relatively flat coastal terrain, having isolated small hills, marshes, and a few rivers, over a height variation between 88 and 235 feet.

From the center of the Sabino Canyon Quadrangle a square 20,000 x 20,000 feet was selected. This terrain comprises four regions, two relatively flat and two quite rugged areas. The rugged terrain contains a number of canyons which cause steep slopes or large height gradients, over a height variation between 2580 and 4350 feet.

Fig. 1 is a redrawing of the portion of the Lehighton Quadrangle that is used as the example of the two procedures. It shows the straight-line approximated contours superposed on the basic map. For the purpose of these experiments the map is taken as the real world, and there is no concern yet with the errors that may be inherent in that map.

#### B. The Contour Tracing Method

The method of straight line approximations may be applied to any curved lines representing constant values such as height or reflectivity. In the case of natural terrain, the height between two approximating lines is obtained by interpolation. In cultural areas (buildings) the height between contours (edges of buildings) is constant; therefore no interpolation is needed. In this case the lines are not necessarily approximations since most building outlines are made of straight-line segments. For areas of constant reflectivity the boundaries require approximations, but no interpolation is required between the lines representing two different reflectivities.

In any case, positions of the vertices obtained from the straight-line approximations must be rounded off to correspond to the grid system chosen to store the information. Described below are the algorithms adopted for derivation of the binary representation of terrain height using the contour tracing method (Ref. 2). The rules are applied to derive the representation of a sample region.

The flow chart of this procedure, Fig. 2, is central to the discussion. Tables 1, 2, and 3 illustrate the format used in this procedure and contain the binary representation for the example shown in Fig. 1.

It is necessary to define certain items in the description. A selected contour is a contour to be approximated. The elevation of the approximation is the same as the selected contour. A limiting contour is a contour with an elevation equal to the selected contour plus or minus the allowable elevation error for the approximation. (For the example region, selected contours were chosen at an interval of 100 feet, and the allowable error was set at  $\pm 40$  feet.)

An annulus of a selected contour is defined as the area bounded by the lower limiting contour and the upper limiting contour, if they both exist. At summits the area bounded by a lower limiting contour may not contain the corresponding upper limiting contour and may or may not contain the selected contour. At depressions or hollows the area bounded by an upper limiting contour may not contain the corresponding lower limiting contour and may or may not contain the selected contour. An annulus is simply connected when such conditions occur -- when all points within one of the limiting contours belong to the annulus. In all other cases, the annulus is multiply connected (i.e. the annulus is bounded by more than one closed curve).

#### 1. Extracting Data From the Map

As shown in Fig. 2 Part 1, the area of terrain to be approximated is divided into square regions by drawing the outlines of the regions on the map (See Table 1). The annulus of each of the selected contours in the area to be approximated is manually outlined. The limiting contours are determined and are traced with a colored pencil on the outside of the annulus. In other words, the colored lines outline the annulus in such a way that the original limiting contour lines are still visible. The annuli are outlined with different colored pencils in order to distinguish between consecutive or adjacent annuli. The tracing is extended beyond the area boundaries or region boundaries.

The annuli bordering on the corners of the area or region are outlined also, since they are required to determine

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TABLE 1

REGION REPRESENTATION BY STRAIGHT-LINE APPROXIMATIONS TO CONTOURS

MAP:

Quadrangle LEWISTON, PA.  
 Contour Interval  
 Index 100  
 Intermediate 20  
 Scale 1:24,000  
 1 inch = 2,000 feet

GRID LINE SPACING:

Inches 0.5  
 Equivalent Feet 1,000

REGION:

Size: 3.5 inches x 3.5 inches  
 Center:  
 x = 3.5 inches  
 y = 3.5 inches

LIMITING CONTOURS:

Plus and Minus 40 feet

NUMBER OF BITS FOR THE REGION:

Common Data 56 bits  
 Contours 570 bits

the heights of the corners. After all the annuli are outlined in the area or region an annulus is chosen at or near a corner or side of the region to begin the approximation procedure. Each annulus is considered separately in turn.

Data extraction involves the examination of the annuli and the construction of straight-line segments to approximate the selected contours. If the annulus or domain is not simply connected, the procedure of Fig. 2 Part 2 is performed, illustrated by the example of Fig. 3. If the annulus is doubly connected, examine the annulus first for S-curves and then for C-curves:

- If the annulus contains one or more S-curves, draw the double tangent to lower limiting contour line for each of the S-curves. Draw straight lines within the annulus starting at the end points of the tangent lines until the annulus is approximated completely by straight line segments.
- If the annulus does not contain an S-curve, but contains a C-curve, draw a tangent line. Draw straight lines within the annulus starting at the end points of the tangent line until the annulus is approximated completely by straight line segments.
- If the annulus contains no S-curves or C-curves, draw the maximum straight line within the length of the annulus. Complete the rest of straight-line approximation starting at the end.

If the annulus is triply or more connected, draw slits or lines between the limiting contours inside the domain, to reduce the domain to being doubly connected. Treat the slits or lines as part of the upper or lower limiting contour as the case may be and consider as being doubly connected as above.

If the annulus or domain is simply connected, the procedure of Fig. 2 Part 3 is performed, also illustrated by the example of Fig. 3. (The area bounded by the limiting contour and the edges of the map is simply connected.)

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After the straight lines are drawn for one annulus or domain, the distances are measured from the origin to the intersection of the straight lines. The selected contour elevation and the locations of the intersections are recorded as depicted by Fig. 2 Part 4. (See Table 2.)

The measured values are converted to the grid coordinate system of the region. (See Table 2.) The converted value is rounded up or down in order to keep the point within the annulus; the direction of rounding is toward the selected contour.

The next annulus or domain adjacent to the one completed is chosen. The foregoing steps are repeated until all annuli or domains have been considered. This completes the procedure for data extraction.

#### 2. Derivation of the Region Representation

As shown in Fig. 2 Part 5, the derivation of the binary region representation is composed of four operations: Calculation of the terrain height at the corners of region, deletion of unnecessary contours, construction of the contour enclosure tree, and the conversion of all data for the region to binary form. (See Table 3.)

The contour enclosure tree is formed as follows:

- The top of the tree or source node is labeled with the selected contour with the lowest elevation.
- A branch or branches are drawn to nodes and labeled with the next contour elevations which are enclosed by the lowest elevation contour.
- A straight line or branch is drawn from the second lowest elevation contour node or nodes to other nodes labeled with the contours that the second lowest elevation contours enclose. This procedure is continued, drawing the branches and labeling the nodes, until the highest elevation contour or peak is reached and used as a label. (Each node represents a contour.)

TABLE 2a

Contour Data in Decimal Form

Quadrangle LEHIGHTON, PA. Origin: x = 3.5, y = 3.5 Page 1 of 2

Contour Name	Contour Elevation	Vertex Number	Measured Coordinates in Inches		Conversion before Round-Off		Vertex in Region Coordinates		Control Eits	
			x	y	x	y	x	y	Closure	Sorting
11000	600	1	1/8	3 1/2	2 1/4	63	2	63	-	-
		2	0	3 7/16	0	61 7/8	0	62	/	-
		3	0	1 7/8	0	21 7/8	0	22	-	-
		4	17/16	1 11/16	16 7/8	30 7/8	17	30	-	-
		5	1 3/4	2 3/8	31 1/2	42 3/4	31	43	-	-
		6	3 1/2	2 3/16	63	57 1/4	63	52	/	/
11200	700	1								
		2								
11100	700	1								
		2								
DELETED	800	1								
		2								
ETC.										

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TABLE 2b

Data Common to the Region in Decimal Form

Quadrangle LEHIGHTON, PA. Origin: x = 3.5, y = 3.5

Region Size

7,000 feet x 7,000 feet = Size A

Region Name

x Coordinate in the Problem Area 435

y Coordinate in the Problem Area 223

Corner Heights

<u>Point</u>	<u>Elevation</u>	<u>Intervals</u>
Height of Reference		
(0,0)	700	$700/20 = 35$
Increments for the Corners		
(0, <u>63</u> )	617	$-83/20 = -4$
( <u>63</u> , <u>63</u> )	600	$-100/20 = -5$
( <u>63</u> , 0)	1,500	$800/20 = 40$

TABLE 3 - Example of  
REGION REPRESENTATION OF BINARY FORM

Quadrangle LEHISTAN, PA. Origin: x = 3.5, y = 3.5

Data Common To the Region

Region Size	x Coordinate in Problem Area	y Coordinate in Problem Area	Height of Reference	Increments for the Corners (0, 12) (12, 12) (12, 0)		Bits
01	0110110011	011011111	00001110011	-0000100	-0000101	+010100
						56

Data for Contours

Contour Name in Tree	Relative Contour Height	Data for Vertices		Bits
0101000000	-0000101	000010111100	00000011111010	
		00000001011000	01000101111000	
		01111110101100	11111111010011	110
0101100000	+0000000	10001010100000	10010010011111	54
0101010000	+0000000	00000000000000	11111110011011	54
ETC.				
				570

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- Next the nodes are labeled with a contour name. The top or source node is labeled 1.0.0...etc. The next level of nodes are labeled 1.1.0... 1.2.0....,etc. The next level of nodes are labeled 1.1.1.0, 1.1.2.0., or 1.2.1.0..., 1.2.2.0....etc. (The label of a node has the prefix of the node of lower elevation to which it is connected.)
- The contour name is recorded for all the contours not deleted.

The control bits for each vertex in a contour are determined as follows:

- Record a 1 for the closure bit if the line segment is not to be considered joining the given vertex with the next vertex in the sequence of vertices for the same contour. This condition occurs when the contour leaves the region and re-enters. Record a 0 if the straight line approximation continues to the next vertex.
- Record a 1 for the sorting bit if the given vertex is the end of the list of vertices for the contour. Record a 0 if the vertex is not the end of the list or sequence.

This completes the derivation of the region representation.

### 3. Problems and Observations

Some methods are noted here for reduction of the total number of bits by modifying the procedure used to obtain the approximations. The maximum vertical error obtainable from the approximations is described.

As shown by Tables 1 and 3 the region is given in digital form by a total of 626 bits. While this region is unusually complex, one of the factors leading to the large number of total bits is the number of contours. The radar land mass simulator of Refs. 2 and 3 assumed a maximum of seven contours per region. The basic rules described therein were followed and extended where necessary. A glance at the

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map of Fig. 1 shows that additional contours could be deleted without changing the maximum vertical error. For example, the lower right corner is labeled 1500 in Table 1; therefore the 1500-foot contour approximation could have a vertex at this point (63,0) instead of passing through the region. This is a saving of 54 bits by a reduction of one in the number of region contours.

Two vertices or 28 bits could be removed from the 600-foot contour by using the upper left corner (0,63) as a vertex. This shift is in the direction of the 600-foot contour and so decreases the error. However, it may also increase the number of vertices in other regions.

By first adding a vertex to the 1300-foot contour and to the 1400-foot contour two more contours could be deleted by the foregoing procedure, namely the 1200-foot and 1300-foot contours. The net saving is 68 bits (1200) + 54 bits (1300) - 14 bits (one additional vertex to the 1400 foot contour) = 108 bits.

These three simple changes result in a reduction of 190 bits by the deletion of three contours over the original region representation. The general conclusion to the above observations is that the optimal approximation to each contour individually is not the criterion for the minimum number of bits to represent the region. A more complex program is needed to derive approximation lines that are nearly parallel and at the same time remain within the limiting annulus. This would reduce the number of contours per region and perhaps increase the number of vertices per contour. However, as shown above, there is a net reduction in the process.

Another conclusion apparent from the observation is that assignment of region boundaries on the terrain may profitably be performed before the contour lines or vertices are determined. In that manner the corners of the region may be assigned as vertices for contour approximation, saving a contour listing in one or two regions without exceeding the permitted error.

The valley extending diagonally across the upper part of the region is bounded by the 540-foot limiting contour (not shown on Fig. 1). As a matter of fact, the 520-foot contour extends approximately halfway across the region. With

the method presented, this area is considered as elevation 600 feet. Therefore the representation is in error by more than eighty feet in some places, as a result of the deletion of intermediate contours.

Another source of error occurs as a result of the roundoff of the vertices to the nearest grid coordinate. This type of error occurring in the sample region results in a maximum error of 60 feet.

In this sample region the maximum theoretical error, 140 feet vertically, is not approached. Actually the 700-foot contour near the center of the region could be deleted and still allow an interpolated height that is within 100 feet of that given by the original map.

For the sample region and its binary representation the distribution of bits to represent various quantities gives an indication where a saving is needed. Approximately 17% of the bits are required to represent height or relative height. Thus a small saving here is not effective overall. The contour name requires 25% of the bits, and 54% for the vertices. Therefore the greatest saving occurs if the number of vertices is decreased. The procedure outlined above for contour deletion is the most promising direction for this method.

### C. The Fixed Point Method

This method is a new approach, applying the concept of height gradients (in three dimensions) to a portion of terrain. The method uses a series of height differences at fixed grid points. The locations of the (equi-spaced) grid points are known in advance, so they need not be stored, and the sequence in which the heights are tabulated is specified by an angle common to the region to be represented.

The height difference can be encoded to make use of contextual dependencies (propensity for slopes to be clustered or nonuniformly distributed). In addition to the saving in amount of data (number of bits per region) by a choice of axes that leads to efficient encoding, no bits are required for contour location data. The method assumes a fixed grid system of arbitrary orientation but with equally spaced points  $(x,y)$  to be tabulated (only cartesian coordinates used herein) and with quantized height differences between adjacent points.

For example, in Fig. 1 point (1,1) has height  $h = 650$  feet and point (1,3) has height  $h = 550$  feet, where the distance between grid points is the constant 2,000 feet. The height difference is  $\Delta h = -100$  feet, but the grid system is fixed; therefore the second point requires only information concerning the slope in the appropriate direction, i.e.,  $\Delta h / \Delta y = -0.05$ . This value of slope is then represented by a few bits of encoded slope data, and the saving may be appreciable.

The development of the coding scheme for data extracted from a contour map, and the procedure for applying the scheme to the representation of the data for a region of that map, are described below. It is shown how the orientation of the coordinate system may be changed to make slopes cluster in a narrow range along one coordinate, at the expense of the non-clustering of slopes along the other coordinate, with a resultant improvement in the data compression afforded.

The region of the Lehighton Quadrangle used as the example for the contour tracing method is also used as the example for the fixed point method. Note that the coding scheme is developed on the basis of a grid of mesh size 1,000 feet, but the region representation is based on double that mesh size (16 points tabulated).

#### 1. Development of the Region Coding Scheme

A coordinate grid was constructed with spacing  $1/2$  inch corresponding to 1,000 feet. A border 1,000 feet wide was retained around the area to provide contour information for calculating gradients on the region boundary. This square of 10,000 x 10,000 feet thus contained  $11 \times 11 = 121$  points.

At each grid point the distance  $\Delta x$  between two 20-foot contours parallel to the  $x$ -axis was measured, converted to feet, and divided into the 20 feet to obtain the  $x$  component of the gradient  $\Delta h / \Delta x$ . The same procedure was used to obtain the component of the  $y$  gradient,  $\Delta h / \Delta y$ .

For example, consider grid point (6,2) in Fig. 1. (This example uses 100-foot contour information rather than 20-foot contours). The  $\Delta x$  measuring from 700-foot to 800-foot contours along the line  $y=4$  is 0.72 inches or 1440 feet. Thus  $\Delta h / \Delta x = 100 / 1440 = 0.066$ . The  $\Delta y$  from 800-foot to 700-foot contours along the line  $x = 7$  is 0.4 inches

or 800 feet. Thus  $\Delta h / \Delta y = 100 / 800 = 0.125$ .

A singular situation was discovered which called for subjectively estimating the height of the grid point. If a particular grid line intersects one contour twice and no other in between, there is no  $\Delta h$ . However, the points between these two intersections have gradients. Take grid point (5,5) for example. The grid line  $y = 5$  intersects the 600-foot contour twice. An estimate of the height is obtained from the  $x = 5$  grid line as

$$600 + (0.12 / 0.42) 100 = 600 + 28.6 = 629 \text{ feet}$$

where 0.12 inches is the distance from the 600-foot contour to the grid point (5,5) along the grid line  $x = 5$ , and 0.42 inches is the distance from the 600-foot to the 700-foot contour along grid line  $x = 5$ . Assuming the terrain height is increasing in the  $x$  direction at a constant rate, the gradient is

$$\Delta h / \Delta x = (629 - 600) / 0.16 (2000) = 0.09 \text{ and}$$

$$\Delta h / \Delta y = 100 / (0.42 \times 2000) = -0.12.$$

This type of situation occurs at grid point (4,2) except that the grid line  $y = 2$  intersects two different contours of the same height. In summary, this situation requires an estimate of the height, location of the maximum height along the grid line, and a determination of the direction of the slope.

An interesting phenomenon occurs if the grid point is located in a valley or on a plateau, or if one coordinate  $x$  or  $y$  is parallel to a contour. For the case in which the grid point is in a valley or plateau, both gradient components are zero. This is illustrated in Fig. 1, for example, by coordinates (5,7), (6,7), (7,7), and (8,7). If the grid point is located such that either axis is parallel to a contour line, the component of the gradient is zero for that coordinate. These last examples are not obvious from the reproduction of the original map (Fig. 1) because the 20-foot intermediate contours are not shown. The frequency of occurrence of this case would be greatly increased by rotating the axis of the coordinate system to where the axis is parallel to the contours. In the example of 121 grid points, 13 (slightly over 10 per cent) of the points have zero gradients for both coordinates. This result is of course independent of rotation of axes. Approximately 5 points (4 per cent) have one gradient equal to zero for the chosen set of axes. This result is clearly variable with rotation of axes.

The  $\Delta x$ 's and  $\Delta y$ 's were measured as described in the previous paragraphs and the gradients calculated. These gradients are plotted as  $\Delta h/\Delta y$  versus  $\Delta h/\Delta x$  in Fig. 4. This is a form of scatter diagram in which the clustering of components of the gradient is obvious. The 59°-clockwise-rotated coordinate system is shown superposed. Since the gradient is a vector quantity, it may be represented in such an arbitrarily transformed coordinate system.

The rotation of axes was made so as to cause clustering of the  $y$  component of the gradient about  $y=0$ . The non-uniformity of the distribution will be shown below to allow encoding with fewer bits overall. As discussed in the preceding section, the improvement occurs with this rotation because there will be more  $y$  differences tabulated than  $x$  differences. The rotated coordinate system thus found by examination of Fig. 4 is also shown on the map of Fig. 1.

The encoding procedure must be optimum in the sense that the average number of bits for representation of the terrain is minimum. A secondary requirement is the relative simplicity of the decoding equipment. The Huffman code meets both of these requirements. In this code, the bits are assigned according to the probability of occurrence of the event or symbol, i.e., for the terrain application it is the probability of occurrence of a value of the height gradient. The procedure of Huffman coding is explicit and is best illustrated by a simple example.

A hypothetical probability curve of one component of height gradient is shown in Fig. 5a. The gradient intervals  $g_i$  are listed in Fig. 5b in order of decreasing probability, given by the area enclosed between the boundaries.

The procedure is to find the two lowest probabilities (0.05's), then to draw lines from these two to a point which is labeled with the sum (0.1). The sum gives the probability that either one interval or the other occurs. This procedure is repeated, the two lowest probabilities (0.1's) being tied by lines to the sum (0.2). Then lines are drawn (from 0.1 and 0.2, etc.) until the sum is equal to 1.

Starting from the node labeled 1.0 the upper branch is labeled with a binary one and the lower branch is labeled with a binary zero for each of the nodes. The code used for an interval is the sequence of digits encountered from node 1.0 to the event in question. These codes are shown in Fig. 5c.

The representation of the seven height gradient intervals in straight binary form would require three bits. The last column of Fig. 5c shows that the average number of bits required using the Huffman coding is 2.6 bits, or a saving of 13 per cent.

No codeword is a prefix of any other longer codeword. Therefore a sequence of the codewords without spaces can be uniquely decoded into the height gradients. The general method of Huffman coding, used in this illustrative example, is applied below to the sample map area.

The discrete values of components of the gradient for each coordinate were placed in ranges equal to 0.05. (In the illustrative example above the range was 0.1) This results in 15 or fewer ranges to be coded. For 1000-foot spacing between points the 0.05 range results in a representation to the nearest 50 feet, which is a reasonable value for height accuracy. If the spacing is increased, the number of ranges must be increased, i.e., the span of each range interval decreased. Perhaps a variable range would be an advantage, but uniform range intervals were used in this experiment.

In Table 4 of the first column is the center value of gradient for the range listed in the second column. The number of occurrences of the gradient within the range is tabulated in the third column. The occurrences of a gradient range are used in deriving the code in the same way as probability was used in the illustration above. Table 4 is arranged in order of increasing gradients rather than in order of occurrences. This arrangement is more convenient when the gradient is known and the code is desired for a particular point in a region for which a representation is being derived. However, to construct the coding tree it is more convenient to list the occurrences in increasing order and cross off the number of occurrences as the tree is drawn.

The cost for each interval is the product of the number of bits and the gradient occurrences for that code and is used to obtain an average number of bits per gradient representation. At the bottom of Table 4 is a calculation of the saving over an equivalent straight binary representation. In straight (scaled) binary, all intervals must be allowed four bits; with Huffman coding only the needed bits per interval are used -- none for those intervals in which no gradients are found.

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TABLE 4

CODING TABLE FOR 121-POINT EXAMPLE IN ONE COORDINATE

<u>Gradient Interval*</u>	<u>Range Covered</u>	<u>Number of Occurrences</u>	<u>Constructed Codeword</u>	<u>Cost In Bits</u>
-0.25	-0.275	1	110,011,1	7
.	-0.225	.		.
.		.		.
.		.		.
0	-0.025	34	10	68
.	+0.025	.		.
.		.		.
.		.		.
+0.50	+0.475	3	011,11	15
	+0.525			
Total Number of points		121	Total Cost in bits	380

\*16 intervals or 4 bits per interval by straightforward digital representation; cost would be  $4 \times 121 = 484$  bits.

Table 5 shows the compression that was obtained in each direction of slope coding for the example. It was noted above that if an axis was parallel to the general direction of the contour lines, the gradients in the direction of that particular axis would tend to be small or zero. The gradients in the other axis direction tend to become larger, but the net effect is a reduction in the average number of bits. In applying these different gradients the result is a reduction of the bits required to represent a region.

Fig. 1 shows that the rotated region has its y axis parallel to the contours in the bottom of the figure, and therefore the gradients in that direction for the region are very small. Note that the gradient is independent of the coordinate system but the components are dependent on the orientation or rotation of the axes. Therefore the change in distribution of gradient components as a result of rotation can be used to advantage by rotating the axes as shown in Fig. 4. The gradient components from the rotation of coordinates were coded in the same manner as in the original data, and Table 5 shows the increased compression obtained. Note that this is characteristic of the codes derived from region gradient distributions.

The clockwise rotation of  $59^{\circ}$  chosen by scrutiny of Fig. 4 would cause an increase (from 968 to 1,089) in the number of bits for a straightforward binary representation, because of the increased number of intervals to be represented. However, the rotation allows a decrease (from 793 to 686) in the number of bits for the encoded representation, because of the clustering or nonuniformity of the distribution of slope. The rotation is seen to accentuate the effect of selecting the primary scanning line to be in the x or y direction.

## 2. Application to the Data for the Region

The example region to be represented is a square 4 x 4 inches or 8,000 x 8,000 feet as shown in the map area of Fig. 1. The method presented here represents the region by storing encoded heights at fixed grid points, using the coding scheme developed above.

More precisely, a reference height is stored at one point (lower left) and differences are stored for the other points. The quantity to be stored is calculated from the difference in height and distance between two adjacent points along a grid line. Here the mesh size is doubled to 2,000 feet

TABLE 5COMPARISON OF CODE EFFICIENCY FOR ROTATED COORDINATE SYSTEM

<u>Coordinate System</u>	<u>Coordinate Direction</u>	<u>Bits per Interval</u>	<u>Bits for No Coding</u>	<u>Bits with Huffman Code</u>	<u>Compression Ratio</u>
Original	x	4	484	380	0.78
	y	4	<u>484</u>	<u>413</u>	0.85
			968	793	
Rotated	x	5	605	443	0.73
	y	4	<u>484</u>	<u>243</u>	0.50
			1,089	686	

( $4 \times 4 = 16$  points per region). The gradient quantization interval of 0.05 then causes rounding of the data at the grid points to the nearest multiple of 100 feet. A strict procedure was used for obtaining the required data in both orientations, calculating and encoding the gradients at the fixed points, and tabulating the derived binary representation of the region.

In practice the coding scheme would be developed for a number of regions having similar characteristics, and data for each region would be encoded by the same scheme. In this example, however, only one region was used. Data was recorded on height at the 16 grid points, rounded to the nearest multiple of 100 feet in accordance with the quantization of the gradient distribution for the  $x$  and  $y$  directions. This recording of data was facilitated by use of a transparent sheet with ruled grid lines at the proper spacing.

Because height contours are tabulated to 100 feet with intermediate contours at 20-foot intervals, the estimation of height to the nearest 100 feet is quite simple, without making measurements of distances between contours enclosing each grid point. The recording was also done for the region under the clockwise rotation of coordinates of  $59^\circ$ . This round-off procedure allows errors at the grid points that are in addition to the error due to the assumption of uniform slope between grid points.

In the  $4 \times 4$  representation of the region a reference point (lower left) is chosen; its height must be given explicitly. Then the heights of the three points in the same row parallel to the  $x$ -axis are tabulated in the form of successive slopes; these are the secondary reference points. Finally, heights of the twelve remaining points in the four columns are tabulated in the form of slopes with respect to those other four points, as discussed in the preceding section. This is the scanning sequence (Fig. 6) when the  $x$ -coordinate is used as the secondary reference; the same is done for the  $y$ -coordinate, with "row" and "column" interchanged.

Table 6 illustrates the construction of the region representation, for the case of the  $x$ -coordinate being the secondary reference, and the region that of Fig. 1. The three secondary reference points are thus encoded by the coding scheme of Table 4, and the twelve remaining points are encoded by the coding scheme similarly derived for the  $y$ -coordinate.

TABLE 6

REGION REPRESENTATION FOR 16-POINT EXAMPLE REGION

<u>Grid Point</u>	<u>Height Difference</u>	<u>Quantized Gradient</u>	<u>Codeword Derived Previously</u>	<u>Number of Bits Required</u>
$(x,y)$	$\Delta h_x$	$\Delta h_x / \Delta x$		
(3,1)	100	0.05	00	7
(5,1)	100	0.05	00	
(7,1)	200	0.1	111	
	$\Delta h_y$	$\Delta h_y / \Delta y$		
(1,3)	-100	-0.05	00	8
(1,5)	0	0	110	
(1,7)	0	0	110	
(3,3)	-100	-0.05	00	7
(3,5)	-100	-0.05	00	
(3,7)	0	0	110	
(5,3)	-200	-0.1	100	8
(5,5)	0	0	110	
(5,7)	-100	-0.5	00	
(7,3)	-300	-0.15	111,0	10
(7,5)	-200	-0.1	100	
(7,7)	0	0	110	
			Total	40

By virtue of the unique decodability of Huffman coding, the sequence of codewords without spaces can be deciphered: the first three codewords give the gradients for the secondary reference points, and the other twelve codewords give the gradients for the remaining points, in the known sequence.

Table 7 compares the number of bits required for the region representation using both the original and the rotated coordinate systems. In each case there is some additional data needed for the region as a whole, such as its reference height and which coding scheme has been applied.

Table 7 shows a saving in bits required to represent the 15 points of the region if one secondary reference is used rather than the other. For the original coordinates the difference is small, but with the rotated coordinates the saving is significant for the 15 points if the secondary reference is along the x-axis. This is a direct result of the rotation which caused distribution of the y components of the height gradients to peak near the small or zero value. There is a pronounced saving for the rotated region with proper choice of secondary reference (scanning sequence). As noted above, the power of the method lies in choosing the secondary reference scanning line so as to maximize the number of bits required; then the orthogonal scanning lines (Fig. 6) result in four (in this example) times as many points, but of high code efficiency. This may be seen by comparison of Tables 5 and 7 -- using the axis of poor code efficiency as the secondary reference achieves good region representation efficiency.

### 3. Data Compression and Accuracy

The binary representation using this method -- encoded, quantized height gradients at fixed coordinate grid points -- results in a decrease in the number of bits for the example region by an order of magnitude over the contour tracing method. In addition, it allows a reconstruction algorithm to be derived that produces continuity of height across the region boundaries. That is, the height of any point on the terrain may be interpolated from the heights of the four surrounding points that are tabulated as shown above. If a point whose height is to be found lies on a grid line (between two tabulated points), the interpolation rule reduces to a linear interpolation between those two points, regardless of the values of the pairs of points to either side of that pair. This algorithm is developed

TABLE 7

COMPARISON OF REGION REPRESENTATION EFFICIENCY

Coordinate System	Secondary Reference	Number of Bits Required		
		(3) Sec. Ref.	(12) Other Pts.	(15) Total
Original	x	7	33	40
	y	8	26	34
Rotated	x	10	17	27
	y	4	39	43

and applied to typical points to determine the error in the 4 x 4 point representation.

Given the heights of four points in a square (Fig. 7) at coordinates  $(i, j)$ ,  $(i + 1, j)$ ,  $(i, j + 1)$ , and  $(i + 1, j + 1)$ , find the height at some interior point  $(i + p, j + q)$  where  $0 < p, q < 1$ . The height at point  $(i + p, j)$  is linearly interpolated between  $(i, j)$  and  $(i + 1, j)$  as  $h(i + p, j) = ph(i + 1, j) + (1 - p) h(i, j)$  and the height at point  $(i + p, j + 1)$  similarly as

$h(i + p, j + 1) = ph(i + 1, j + 1) + (1 - p) h(i, j + 1)$ . Finally the height at point  $(i + p, j + q)$  is linearly interpolated between these two points as

$$\begin{aligned} H(i + p, j + q) &= qh(i + p, j + 1) + (1 - q) h(i + p, j) \\ &= pqh(i + 1, j + 1) + (1 - p)qh(i, j + 1) \\ &\quad + p(1 - q) h(i + 1, j) + (1 - p)(1 - q) h(i, j). \end{aligned}$$

If the terrain is reasonably smooth, the region can be well represented by the 4 x 4 tabulated points. To check the accuracy, several points were selected with a view to being worst-case conditions for the region. The error for point  $(0.6, 3.4)$  is -14 feet; that at point  $(4.0, 2.4)$  is -50 feet; and that at point  $(0.8, 0.4)$  is -80 feet. These compare with the errors arising from the contour tracing method with 100-foot contour interval. Of course, if the terrain were more rugged, more points would be required for the same accuracy using either method.

The region coordinates and the other common data now require a large fraction of the bits for the contents of the region. Thus a reduction here is worthwhile (i.e., increase the size of the region or reference the small regions to larger regions). The bits required to represent the reference heights are also a large fraction of the total bits. Obviously, the Huffman coding scheme can be applied to them in 4 x 4 groups of regions.

The rotation of the individual region deletes small triangles of area from the map and adds other triangles. No extensive examination of the gradients in these areas was made; however, it is estimated that the net effect for the area considered is an increase in the higher gradients. It is expected that for larger samples this effect would tend to be balanced out. The more important question is, how can the rotation phenomena can be used to the fullest advantage in representing a large area of a map? There are large areas in which

contours and thus slopes tend to be oriented in a particular direction. In order to use different rotations, some overlap of the representation is necessary. However, with high percentage saving for rotation, the net gain can be significant when the rotation is performed over large areas of the map at a time.

#### D. Comparison of Methods for Several Types of Terrain

The preceding procedures were followed to obtain the representations of other portions of terrain. The results using both methods are summarized in Table 8. In each sample the fixed point method requires an order of magnitude fewer bits, regardless of the type of terrain.

In the relatively flat regions of Whitneyville and Sabino Canyon the height gradients required at the grid points are nearly constant. The number of fixed grid points could conceivably be reduced without increasing the error. However, in the Whitneyville regions, the vertical error is larger than by the contour tracing method, since the isolated hills or peaks do not occur at the fixed grid points. Therefore, the representation for this type of terrain might require singularities at a few fixed grid points, with a small increase in bit requirement.

For the contour tracing method the storage requirements are less for flat terrain, because the contour tracing method is more readily adapted to terrain of different types -- contours are placed only where the terrain changes height by a specified amount. If the terrain is rolling with isolated hills and peaks, as in the Whitneyville regions, very few contours can be deleted. If the terrain contains ridges, some reduction is possible as a result of deletion of contours.

In the very rugged terrain of the Sabino Canyon regions the maximum vertical error using the 16 fixed points is very high. One sample point was in error by over 600 feet. The 2,000 feet between grid points can span a high peak or a deep canyon. Therefore, to maintain the vertical error to an acceptable value, the number of fixed grid points must be increased, singularities and ridges must be added as supplementary data, or the method must be improved.

TABLE 8  
COMPARISON OF STORAGE REQUIREMENTS FOR CONTOUR TRACING AND FIXED POINT METHODS

Quadrangle	Coordinates of Region Center	Bits Per Region Contour Tracing Method	Fixed Point Method						Angle of Rotation	
			Original Coordinates		Rotated Coordinates		Reference	Secondary		
			x	y	x	y		x		y
Leighton, Pennsylvania	$\begin{pmatrix} 3.5, 3.5 \\ 3.0, 3.5 \end{pmatrix}$	570	40	34	27	43			-59°	
Whitneyville, Maine	$\begin{pmatrix} 8, 15 \\ 4, 15 \end{pmatrix}$	<div>244 136</div>	<div>15 18</div>	<div>16 17</div>	<div>17 15</div>	<div>19 16</div>			+45°	
Sabina Canyon, Arizona	$\begin{pmatrix} 10, 8-3/8 \\ 10, 12-3/8 \\ 14, 8-3/8 \\ 14, 12-3/8 \end{pmatrix}$	<div>466 2148* 122 4228*</div>	<div>28 55 24 77</div>	<div>24 54 17 66</div>	<div>18 32 46 77</div>	<div>30 29 47 62</div>			+32°	

\*Contour interval 200 feet; elsewhere 100 feet.

## IV. CONCLUSIONS AND RECOMMENDATIONS

It has been shown by the work reported herein that there are two fields in which further development should have a potential effect upon the realism of any map representation and reconstruction process. The field of numerical analysis relates the maximum error of a function approximation to the characteristics of the function being approximated. Continuity conditions can be invoked by suitable restrictions on derivatives of the approximation, and indication can be obtained of the optimum grid and region sizes to be used in order to maintain the error at a specific upper bound by a suitable formalized criterion.

Along with the developments in numerical analysis as applied to functions of many variables, recent developments in information theory indicate methods for obtaining a measure of the information content of a data store. This is important because such a measure represents the theoretical minimum amount of data (in the sense of bits or computer words) that will be required to afford a satisfactory representation and reconstruction of a mapped function, subject to suitably formalized error or accuracy requirements. Coding theory has been applied to the storage of digital map information, and it has been shown that the amount of data for certain representations can be significantly reduced, without any sacrifice in the accuracy or information content of the reconstruction of the originally encoded function. Information theory also provides techniques for determining the actual information content of raw data sources, including photography, radar and infrared records, other maps, and geodetic and intelligence data.

All such methods and algorithmic techniques imply trade-offs between the amount of information contained in the storage medium and the complexity of processing to reconstruct the radar view of the real world. Encoding to minimize storage implies more complicated processing to decode the stored data; conversely, organizing the data into independent packets of regions of the real world for convenience of manipulation implies larger storage capacity to account for the added redundancy or lowered efficiency of the static representation. It is recommended that such approaches be pursued further to determine the amount and format of data contained in an optimally arranged simulator data store, based on the approaches investigated both in previous work on radar land mass simulation and in the preliminary work on digital representation of maps reported herein.

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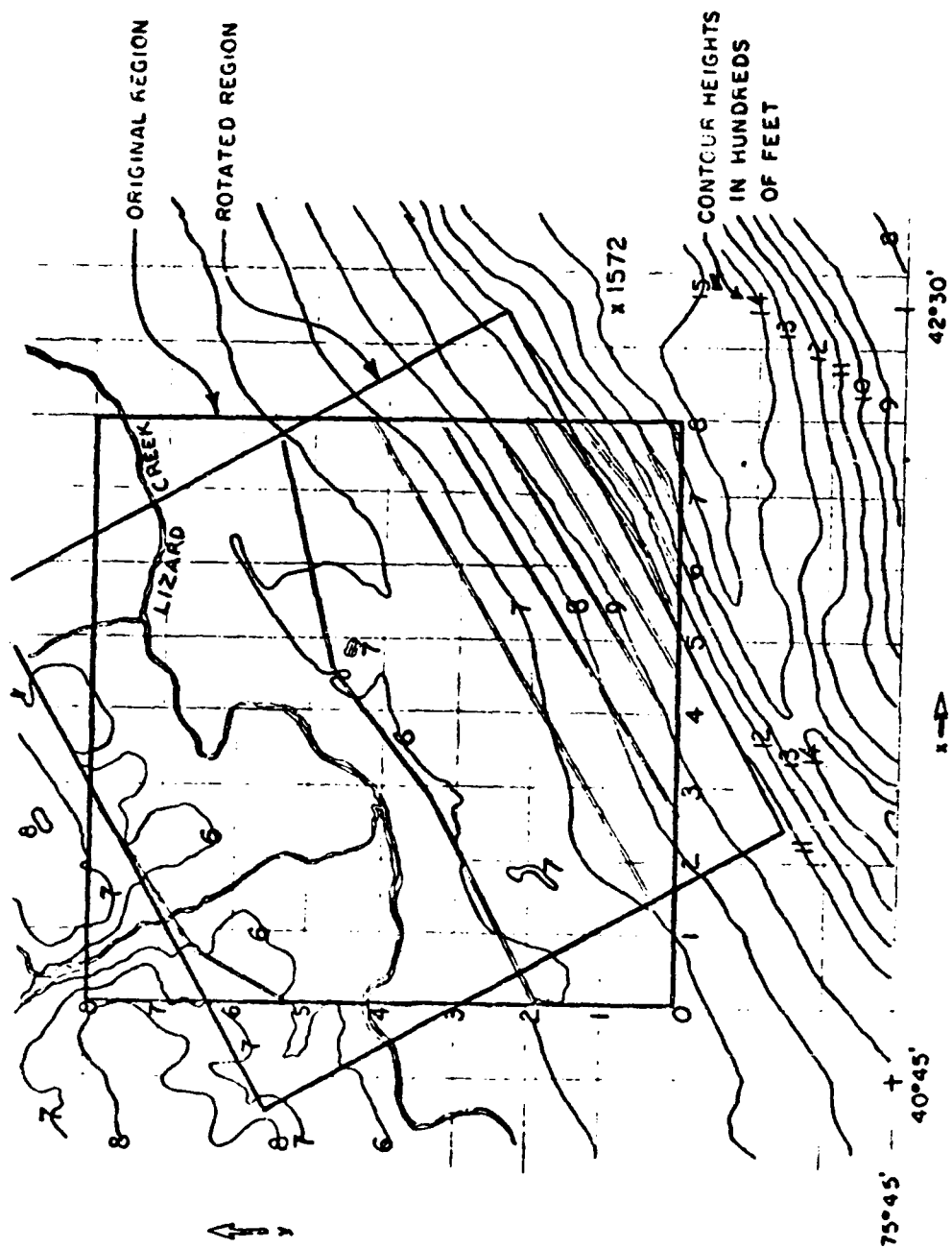


FIG. 1 - SECTION OF LEIGHTON QUADRANGLE USED AS EXAMPLE

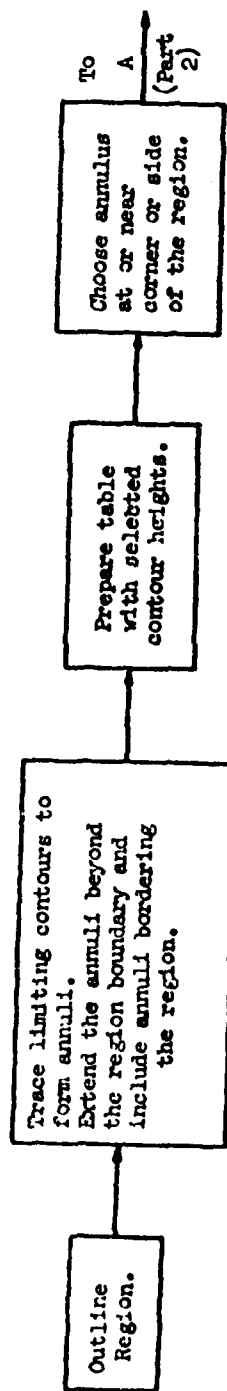


FIG. 2 - PROCEDURE FOR CONTOUR REPRESENTATION IN BINARY FORM (PART 1 OF 5)

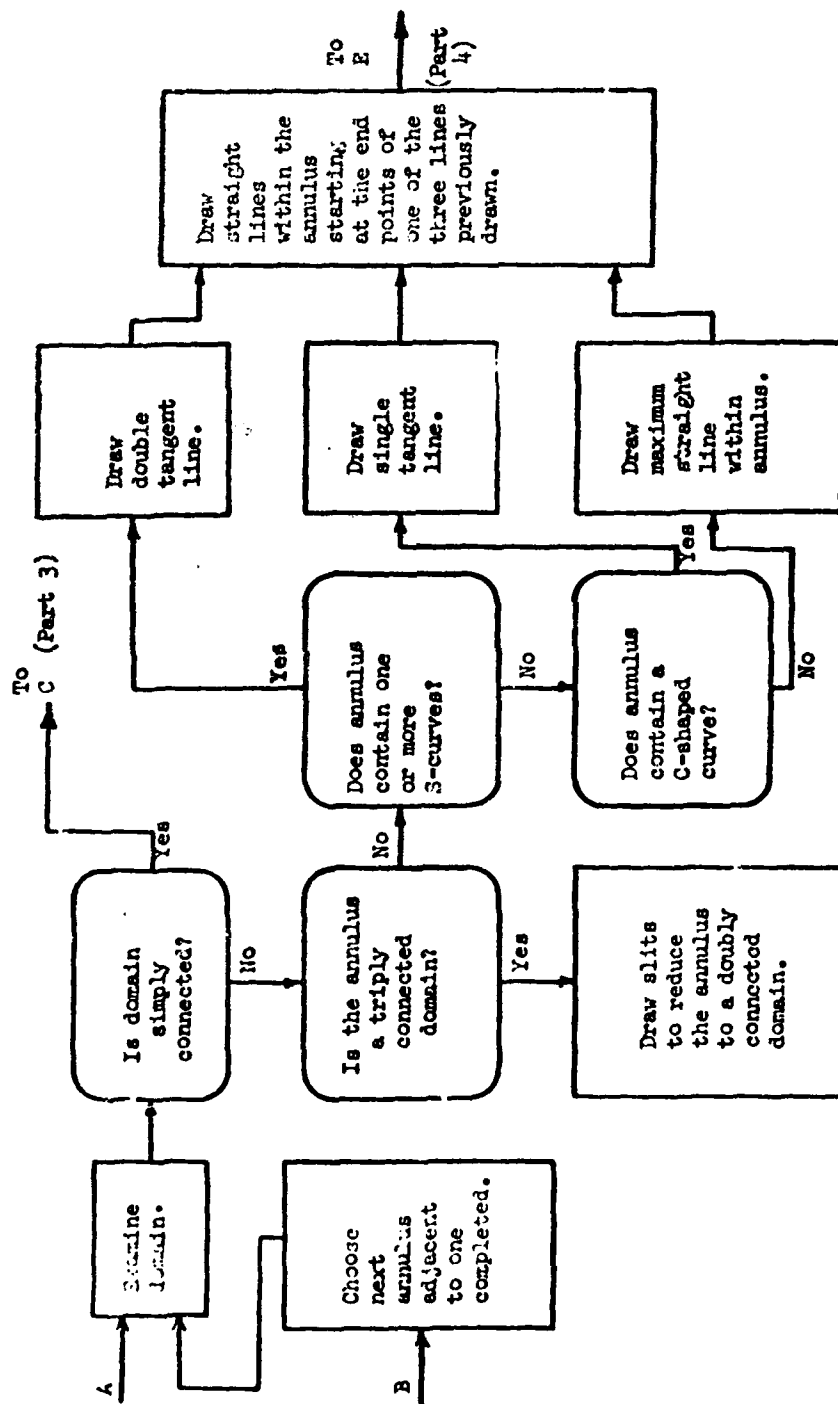


FIG. 2 - PROCEDURE FOR CONTOUR REPRESENTATION IN BINARY FORM (PART 2 OF 5)



**FIG. 2 - PROCEDURE FOR CONTOUR REPRESENTATION IN BINARY FORM (PART 3 OF 5)**

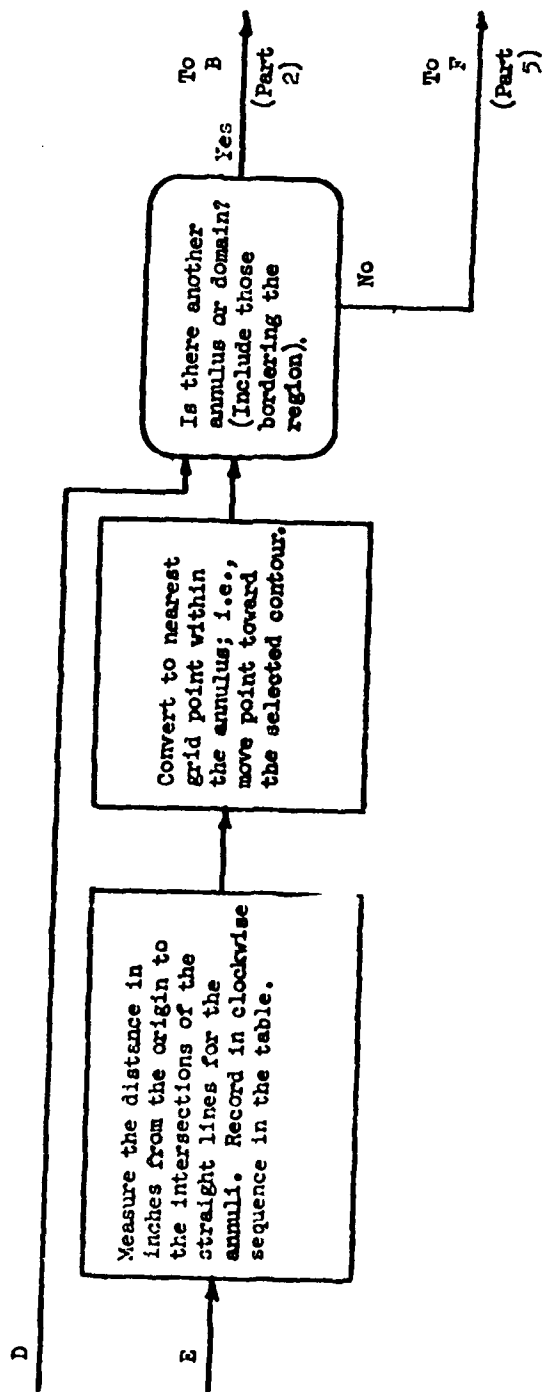


FIG. 2 - PROCEDURE FOR CONTOUR REPRESENTATION IN BINARY FORM (PART 4 OF 5)

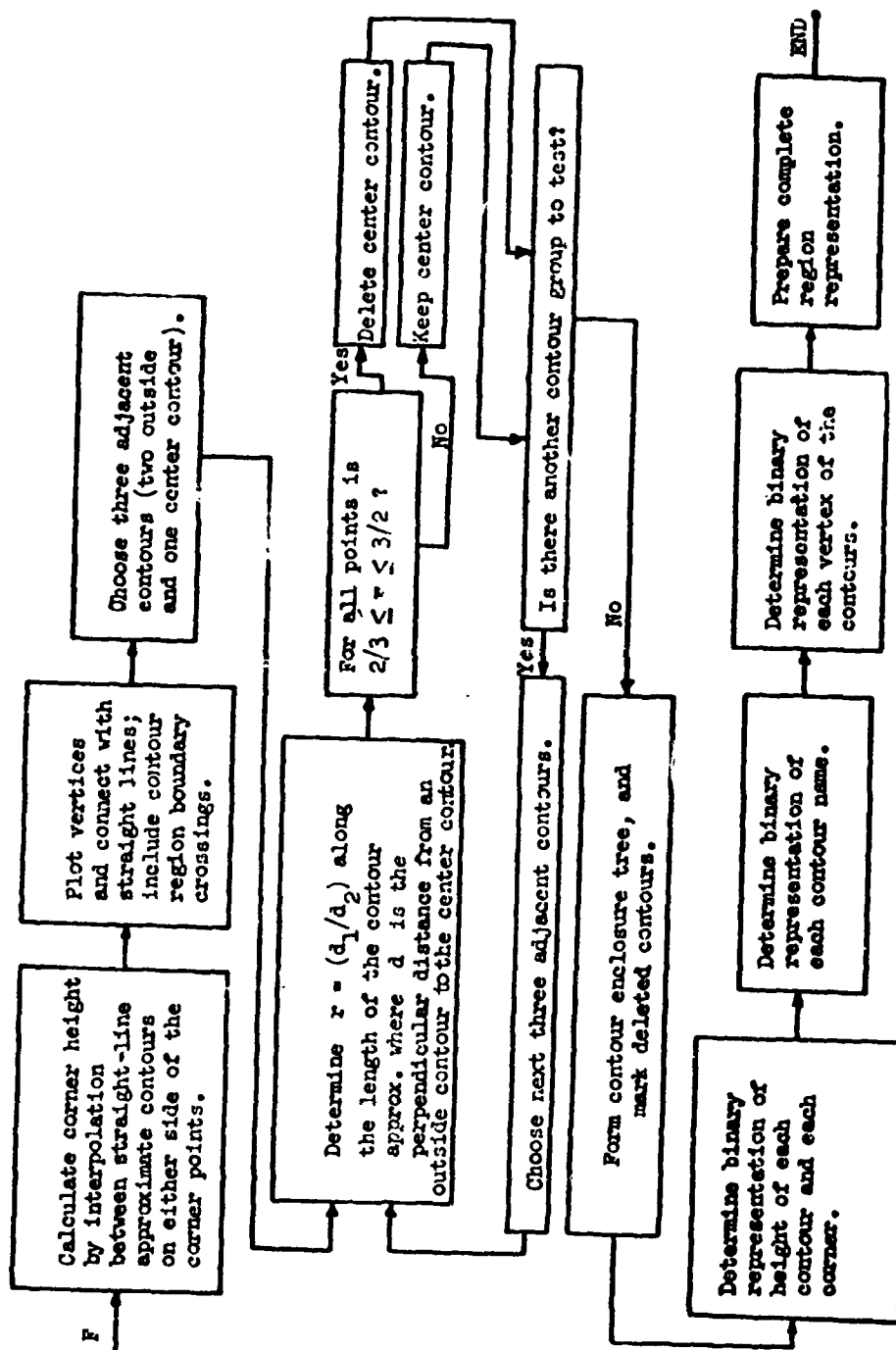


FIG. 2 - PROCEDURE FOR CONTOUR REPRESENTATION IN BINARY FORM (PART 5 OF 5)

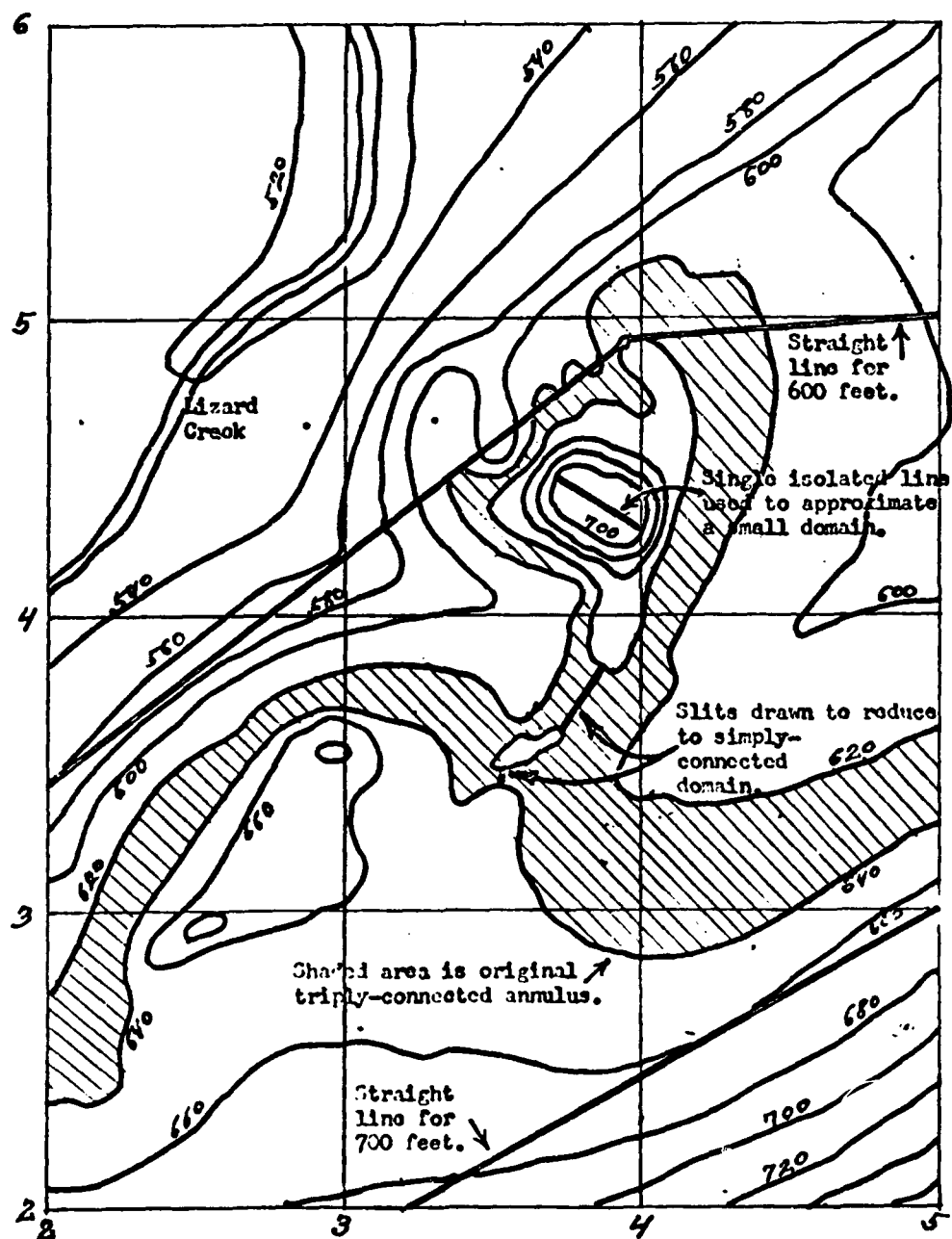


FIG. 3 - EXPANDED PORTION OF THE REGION USED AS EXAMPLE

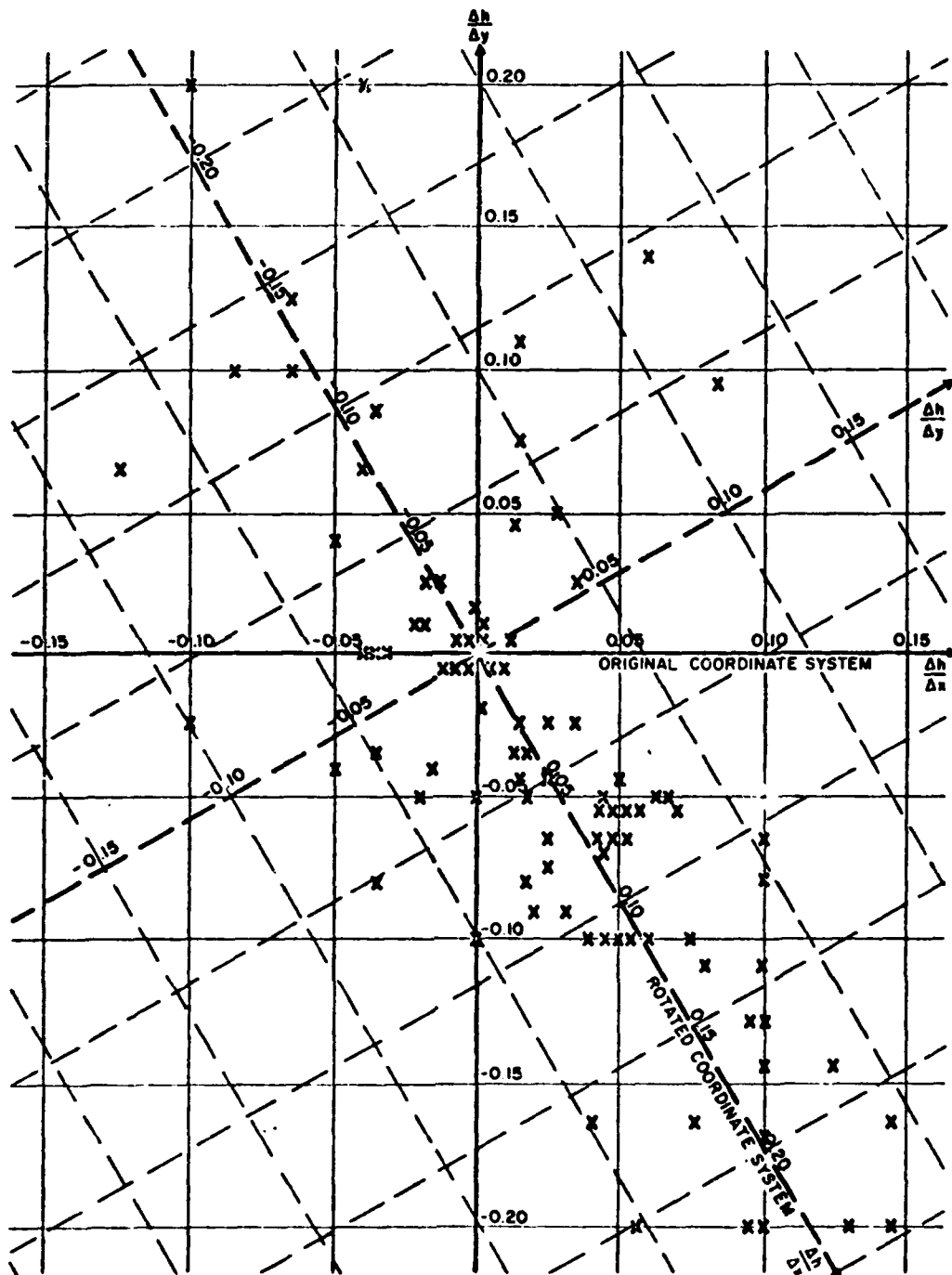
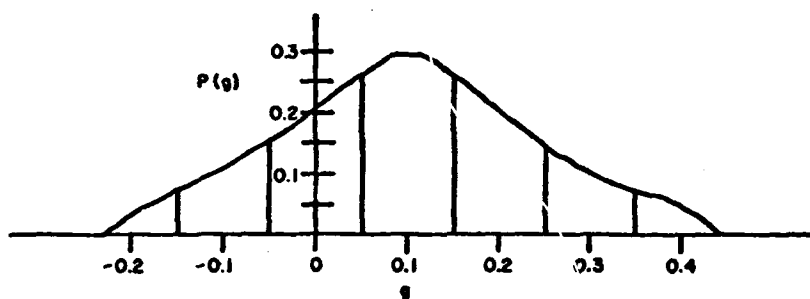
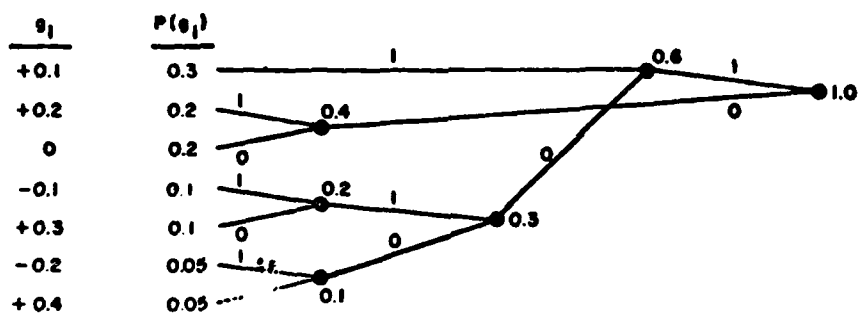


FIG. 4 - PORTION OF GRADIENT DISTRIBUTION FOR EXAMPLE REGION



(a) QUANTIZED PROBABILITY DISTRIBUTION OF GRADIENT



(b) CODING TREE CONSTRUCTED FROM THE DISTRIBUTION

GRADIENT INTERVAL $g_i$	PROBABILITY $P(g_i)$	CONSTRUCTED CODEWORD	NUMBER OF BITS IN CODE, $N$	COST $N \times P(g_i)$
0.1	0.3	11	2	0.6
0.2	0.2	01	2	0.4
0	0.2	00	2	0.4
-0.1	0.1	1011	4	0.4
0.3	0.1	1010	4	0.4
-0.2	0.05	1001	4	0.2
0.4	0.05	1000	4	0.2
	1.00			

AVERAGE NUMBER OF BITS PER  
GRADIENT INTERVAL  $\sum_i N \times P(g_i) = 2.6$

(c) CODEWORDS CONSTRUCTED FOR THE GRADIENT INTERVALS

FIG. 5 - ILLUSTRATION OF HUFFMAN CODING OF GRADIENTS

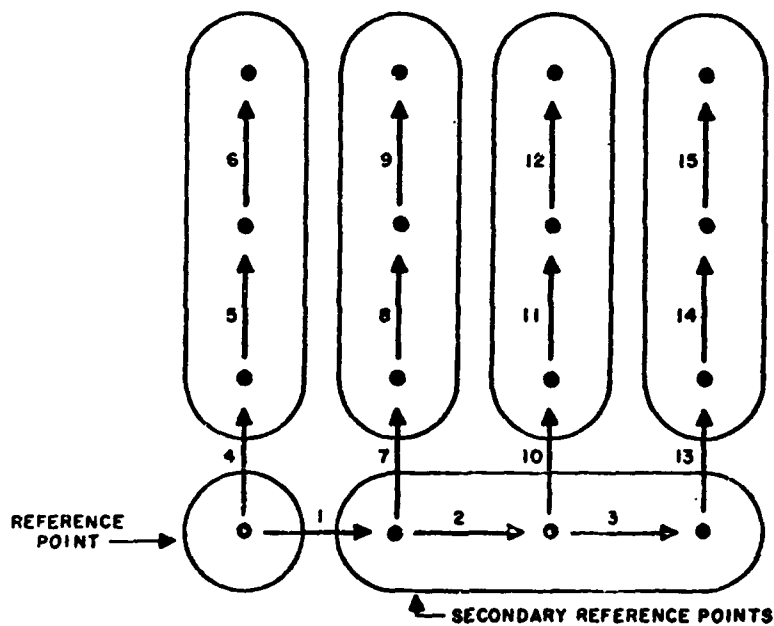


FIG. 6 - SCANNING SEQUENCE FOR 4 x 4 POINT GRADIENT REPRESENTATION

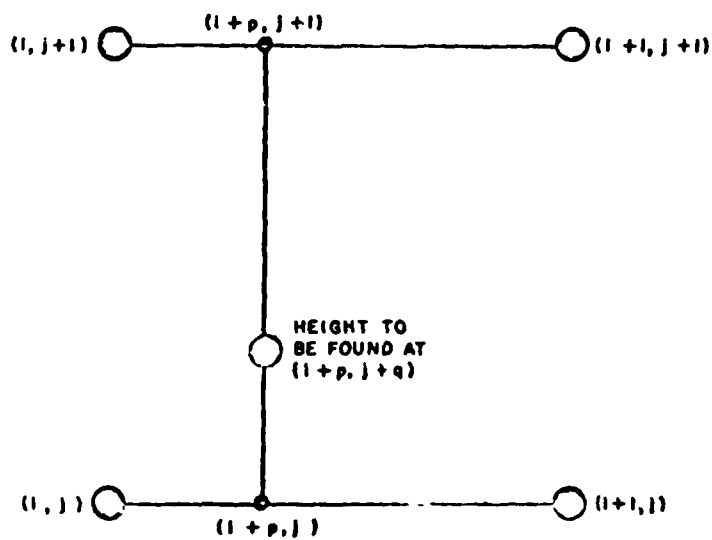


FIG. 7 - INTERPOLATION OF HEIGHT AT AN ARBITRARY POINT

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**NOTICE**

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligations whatsoever; and the fact that the Government may have formulated, furnished, or in any other way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

US NAVAL TRAINING DEVICE CENTER

Apr. 1964

Tech. Rept. NAVTRADEVGEN 1025-4

UNCLASS.

FINAL REPORT: INVESTIGATION OF THE COMPILATION  
OF DIGITAL MAPS

Pennsylvania Research Associates, Inc. (Contract  
N61,39-1025) vi, 46 pp., 7 illus., 8 tables,  
48 refs.

This is the report on a preliminary study to  
determine optimal methods of representation of  
maps in digital form, for use in radar landmass  
simulation and for other training purposes. Some  
techniques of information theory are applied, and  
the vector gradient method of terrain representation  
is derived. Results of experimentally representing  
and reconstructing the terrain by this and the  
contour tracing method are discussed.

KROS-TERMS

COMPRESSION  
COMPUTER  
CONTOURS  
DIGITAL  
GRADIENT  
INFORMATION  
LANDMASS  
MAPPING  
MEASURE  
RADAR  
SIMULATION  
THEORY  
VECTOR